MATEMĀTIKA ANGLISKI

mācību stundu piemēri

Atbildīgā par izdevumu Rita Kursīte

English For Mathematics
Lesson 1
Algebraic Expressions and Equalities (Algebriskas izteiksmes un vienādības)

Mērķis:

1. Iepazīstināt skolēnus ar angļu valodas lietojumu matemātikā.
2. Atkārtot vienkāršas matemātiskas pamatsakarības un izteiksmju identiskos pārveidojumus.

Skolēnam sasniedzamais rezultāts:

1. Saprot vienkāršu matemātisku tekstu par vienādībām angļu valodā.
2. Prot izklāstīt/paskaidrot vienkāršākos algebriskos pārveidojumus angļu valodā.

Nepieciešamie resursi:

1. Teksti „Reading the Symbols”, „Expressions and Equalities” ar uzdevumiem risināšanai
2. Jauno vārdu (matemātikas terminu) saraksts ar tulkojumiem, vārdnīca.

Mācību metodes:

1. Darbs ar tekstu.
2. Jautājumi un atbildes.

Mācību procesa organizācijas formas:

1. Frontāls darbs.
2. Pāru darbs.

Vērtēšana: Skolotājs dod frontālu mutisku novērtējumu.

Stundas gaita: Sniegtais plāns ir samērā elastīgs, lai skolotājs to varētu pielāgot savam un skolēnu matemātiskās izpratnes līmenim. Iespējams apjēgšanas daļu variēt un nedaudz pagarināt uz refleksijas rēķina.

Stundas piemērs

Skolotāja darbība	Skolēnu darbība
Metode (laiks)	
Ierosināšana (5 min). Skolotājs rakstiski un mutiski piedāvā dažādus matemātikas terminus angļu valodā un uzaicina skolēnus mēģināt tos pārtulkot latviešu valodā. Kļūdas/atbildes trūkuma gadījumā skolotājs sniedz pareizo atbildi. Vēlamais secinājums: pamatojoties uz jau esošām angļu valodas un matemātikas zināšanām, daudzus terminus var viegli saprast.	Skolēni nosauc iespējamos tulkojumus.
Metode (laiks)	
Apjēgšana. (20 min) 1. Skolotājs uzaicina skaļi lasīt tekstu ***Reading the Symbols***. 2. Darbs ar tekstu. Skolotājs uzaicina skolēnus lasīt tekstu ***Expressions and Equalities.*** 3. Uzdevumu risināšana. Skolotājs aicina skolēnus izlasīt pirmā vingrinājuma uzdevumu un piemērus; pēc tam izpildīt vingrinājumu. Norāde: katram piemēram ir iespējami dažādi pamatojumi. Pēc skolotāja ieskatiem (un atkarībā no laika iespējām) var pildīt arī otro vingrinājumu vai arī atsevišķus piemērus no abiem.	1. Skolēni pa vienam nolasa izteiksmes un vienādojumus simboliskajā pierakstā. 2. Skolēni lasa tekstu. 3. Skolēni vispirms pieraksta atbildes, pēc tam visi kopā tās salīdzina, t.sk. dažādos pamatojumus.
Metode (laiks)	
Refleksija. (5+10 *min*) Sadalot skolēnus pa pāriem, lūdz pašiem izveidot vienkāršus uzdevumus (t.i., jautājumus par tekstu, uzdevumus par identitāšu pārbaudīšanu vai izteiksmju pārveidojumiem) un uzdot tos klasesbiedriem. Uzdodot uzdevumus klasei, skolotājs vada procesu atkarībā no atlikušā laika un piemēru līdzības (t.i., cenšoties saglabāt jautājumu daudzveidību). Laika trūkuma gadījumā sastādīšanu un/vai atrisināšanu var uzdot mājas darbā.	Katrs pāris sagatavo dažus (vismaz 2) pēc iespējas atšķirīgus jautājumus vai uzdevumus par aplūkoto tēmu, kā arī to atbildes. To skaits būs atkarīgs no skolēnu spējām. Jautājumus uzdod mutiski, ja nepieciešams, uzrakstot tos arī uz tāfeles. Atbildes sniedz mutiski.

Skolotāja pašnovērtējums: Secinājumi par skolēnu spēju ātri apgūt jaunu specifisku terminoloģiju, kā arī stundas atbilstību skolēnu matemātiskajām spējām stundas specifikas dēļ jāveic nepārtraukti stundas gaitā, lai atbilstoši situācijai veiktu korekcijas - pielāgotu gan pašreizējās stundas materiāla un uzdevumu izmantošanu, gan nākamo stundu materiālu un uzdevumu grūtību.

Reading the Symbols

Read these symbols:

$=$	is equal to; equals
$+$	plus
$-$	minus
$\cdot$, $\times$	times; [multiplied] by
$:$, $\div$	divided by
$(\ )$	brackets; parentheses
$\frac{3}{5}$	three fifths; three over five
4.7	four point seven
0.2	zero point two; oh point two

*(Note that in English speaking countries a **decimal point** is mostly used instead of a comma.)*

-5	minus five; negative five
a^b	*a* to the power *b*
a^{10}	*a* to the tenth
a^2	*a* squared
a^3	*a* cubed
$\sqrt{a}$	the square root of *a*

Read the following expressions and equalities.

1. $-7 \cdot (-2) = 14$
2. $2.8(x-3) = 5.6 - 2x$
3. $8x^3$
4. c^5
5. 10^{2x}
6. $x^2 + 3x - 7 = 0$
7. $\sqrt{5}$
8. $\sqrt{x+y}$
9. $\sqrt{2x+1} = 11$
10. $\frac{a+b}{2}$
11. $\frac{y-3}{2} = \frac{y}{4}$

Reading the Symbols
Teacher's Notes

Read these symbols:

$=$	is equal to; equals
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a^2	*a* squared
a^3	*a* cubed
$\sqrt{a}$	the square root of *a*

Read the following expressions and equalities

1. $-7\cdot(-2)=14$ — (minus 7 times minus 2 equals 14; negative 7 multiplied by negative 2 is equal to 14; the product of minus 7 and minus 2 is 14)
2. $2.8(x-3)=5.6-2x$ — (2.8 **times** *x* minus 3 equals 5.6 minus two *x: in written form the multiplication dot before brackets is omitted but still must be read; or, alternatively*: 2.8 brackets open *x*-3 brackets closed)
3. $8x^3$ — (8 *x* cubed *or* 8 *x* to the third)
4. c^5 — (c to the fifth; c to the power 5)
5. 10^{2x} — (10 to the power 2*x*)
6. $x^2+3x-7=0$ — (*x* squared plus 3 *x* minus 7 is equal to 0 *or* equals 0)
7. $\sqrt{5}$ — (square root of 5)
8. $\sqrt{x+y}$ — (square root of *x* plus *y*)
9. $\sqrt{2x+1}=11$ — (square root of 2 *x* plus 1 is equal to 11 *or* equals 11)
10. $\frac{a+b}{2}$ — (*a* plus *b*, over 2; *or a* plus *b*, divided by 2)
11. $\frac{y-3}{2}=\frac{y}{4}$ — (*y* minus 3 over 2 is equal to *y* over 4)

Expressions and Equalities

What is the difference between ***expressions***, ***equalities***, ***identities*** and ***equations***?

An ***expression*** is any combination of ***numbers, variables*** and ***mathematical operations*** connecting them.

Examples of expressions are

$3-5n$

$2^x \cdot 24$

a^2+b^3

An ***equality*** consists of two expressions connected by the sign of equality $=$.

Examples of equalities are

$2+3=5$

$2x+x^2y=6y-1$

An equality is called ***identity*** if it is true for **all** values of variables for which both sides are defined.

Some examples of identities are

$0x=0$

$2(a+3b)=2a+6b$

$$\frac{x^2+2x}{x}=x+2$$

The last identity is true for all real values of x except 0 because the left side is not defined if x=0; division by 0 is impossible.

Exercise 1

For each of the given equalities, state whether it is an identity. Justify your answer.

Examples

A $2(a+3b)=2a+6b$ is an identity. It can be verified by opening the brackets.

B $x^2-4=\left(x-2\right)^2$ is not an identity. Both sides are not equal, e.g., for $x=0$.

1. $\frac{2}{3}x+1=5$
2. $a+7=7+a$
3. $x-12=12-x$
4. $x(5-y)=5x-y$
5. $5a-b-5b-3a=2a-6b$
6. $m^2 \cdot m^3 = m^5$

Exercise 2

Write and simplify (use the brackets where necessary).

1. the difference of $x-7$ and $5-x$

2. the product of $6ax$ and $0.5a^3x$

3. the sum of y^2+3 and $3y^2-2$

4. the quotient of a^2-4 and $a-2$

5. the square of $3b-\frac{2}{3}$

Expressions and Equalities

Teacher's Notes

Exercise 1

For each of the given equalities, state whether it is an identity. Justify your answer.

Student material	Answer and comment for the teacher
Examples A $2(a+3b)=2a+6b$ is an identity. It can be verified by opening the brackets.	The coefficient 2 must be multiplied by each term in the brackets. **Note** It cannot be verified by checking a few or even 'many' values of the variables because **all** possibilities cannot be checked one by one.
B $x^2-4=(x-2)^2$ is not an identity. Both sides are not equal, e.g., for $x=0$.	Any number can be used instead of 0; the only *x* value for which this equality is true is $x=2$; still, it is not enough to claim an identity – for that, the equality must hold true for **all** values of *x*, not just for $x=2$.
1. $\frac{2}{3}x+1=5$	**Not an identity**; easily checked by using any *x* value. The **only** value for which the equality is true is $x=6$. (Encourage the students to solve this equation to demonstrate it.)
2. $a+7=7+a$	**An identity.** The sum does not change if the sequence of terms is changed.
3. $x-12=12-x$	**Not an identity.** The left and right sides are opposite numbers: or, check by letting *x* be any number (except 12).
4. $x(5-y)=5x-y$	**Not an identity.** When opening the brackets, we should get *5x-xy*. Or use any values to check (except $x=1$ or $y=0$).
5. $5a-b-5b-3a=2a-6b$	**An identity**, verified by combining the like terms: $\underline{5a}\ \underline{\underline{-b}}\ \underline{\underline{-5b}}\ \underline{-3a}=\underline{5a-3a}\ \ \underline{\underline{-b-5b}}=2a-6b$
6. $m^2\cdot m^3=m^5$	**An identity**, according to properties of powers: $m^2\cdot m^3=(m\cdot m)\cdot(m\cdot m\cdot m)=m\cdot m\cdot m\cdot m\cdot m=m^5$; or, $a^k\cdot a^n=a^{k+n}$

Exercise 2

Write and simplify (use the brackets where necessary).

Student material	Comment
1. the difference of $x-7$ and $5-x$	$(x-7)-(5-x)=x-7-5+x=2x-12$ - attention should be paid to signs when opening the brackets
2. the product of $6ax$ and $0.5a^3x$	$6ax\cdot 0.5a^3x=3a^4x^2$ (3 *a* to the 4th *x* squared)
3. the sum of y^2+3 and $3y^2-2$	$y^2+3+3y^2-2=4y^2+1$
4. the quotient of a^2-4 and $a-2$	$\frac{a^2-4}{a-2}=\frac{(a+2)(a-2)}{a-2}=a+2$; not defined for *a*=2
5. the square of $3b-\frac{2}{3}$	$\left(3b-\frac{2}{3}\right)^2=9b^2-4b+\frac{2}{3}$ according to the formula $(a-b)^2=a^2-2ab+b^2$

Stundas piemērs

English For Mathematics
Lesson 2
Solving Equations (Vienādojumu risināšana)

Mērķis:

1. Iepazīstināt skolēnus ar angļu valodas lietojumu matemātikā.
2. Atkārtot algebras pamatvienādojumu risināšanas metodes.

Skolēnam sasniedzamais rezultāts:

1. Saprot vienkāršu matemātisku tekstu par vienādojumiem angļu valodā.
2. Prot izklāstīt/paskaidrot vienkāršākos algebrisko vienādojumu atrisinājumus angļu valodā.

Nepieciešamie resursi:

1. Teksti „Solving Equations" , „Who Invented Algebra?" ar uzdevumiem risināšanai
2. Jauno vārdu (matemātikas terminu) saraksts ar tulkojumiem, vārdnīca.
3. Uzdevumu atrisinājumi (skolotājam).

Mācību metodes:

1. Darbs ar tekstu.
2. Jautājumi un atbildes.
3. Izskaidrojums.

Mācību procesa organizācijas formas:

1. Frontāls darbs.
2. Darbs grupās.

Vērtēšana: Skolēni veic sava darba izvērtējumu.

Stundas gaita: Piedāvātais plāns ir ar nolūku veidots tā, lai skolotājs to varētu pielāgot savam un skolēnu matemātiskās izpratnes līmenim. Iespējams apjēgšanas daļu variēt. Visvienkāršākajā gadījumā skolēni iepazīstas ar tekstu 'Solving Equations' un pašiem vienkāršākajiem piemēriem un uzdevumiem; maksimālajā variantā aplūkotie piemēri ir daudzveidīgāki, papildus teksts 'Who Invented Algebra?'. Darba grupas lielums ir 2-4 cilvēki; ja klasē skolēnu daudz, var veidot grupas ar vienādiem darba uzdevumiem.

Stundas piemērs

Skolotāja darbība	Skolēnu darbība
Metode (laiks)	
Ierosināšana (5 min). Skolotājs uzaicina skolēnus atcerēties, ko mācījušies iepriekšējā stundā, pēc iespējas lietojot jaunapgūtos matemātikas terminus.	Skolēni brīvā formā cenšas pastāstīt, ko darījuši, kas vislabāk palicis prātā vai kas sagādājis grūtības.
Metode (laiks)	
Apjēgšana. (25 min) 1. Darbs ar tekstu. Skolotājs uzaicina iepazīties ar tekstiem ***„Solving Equations”*** un ***„Who Invented Algebra?”.*** 2. Skolotājs uzdod jautājumus, lai pārbaudītu teksta izpratni. Jautājumu sarežģītība/konkrētums atkarīgs no klases. 3. Uzdevumu risināšana. Skolotājs sadala skolēnus grupās pa 2-3 un katrai grupai piedāvā izpētīt dotos vienādojumu risinājuma piemērus un iepazīstināt ar tiem klasi. Atkarībā no skolēnu izpratnes līmeņa D grupu (visgrūtākie piemēri) var neveidot un šo uzdevumus neaplūkot. Piemēru matemātiskā grūtība pieaug no A līdz D.	 1. Skolēni lasa tekstu, ja nepieciešams, lieto vārdnīcu vai piedāvāto *Glossary* lapu. 2. Skolēni saviem vārdiem atbild uz jautājumiem. Skolēni var arī paši viens otram uzdot jautājumus. 3. A grupas(-u) skolēni izlasa un apspriež 1. piemēru (*Example 1*); papildus var izmantot 1., 2. uzdevumu vai izdomāt līdzīgus. B grupas skolēni – 2. piemēru un 3. uzdevumu C grupas skolēni – 3. piemēru un 4., 5. uzdevumu D grupas skolēni – 4.,5. (vai tikai 4.) piemēru un 6.,7.,8. uzdevumu. Katras grupas pārstāvis īsi iepazīstina klasi ar savu vienādojuma veidu, komentējot risinājumus vārdiski. Ja laiks atļauj, var uzaicināt citu tematisko grupu pārstāvjus izstāstīt risinājumu uzdevumam no vingrinājuma.
Metode (laiks)	
Refleksija. (10 *min*) Skolotājs aicina skolēnus īsi novērtēt savu darbu, aplūkojot pa tematiskajām grupām vai pēc kārtas individuāli.	Skolēni analizē savu un klasesbiedru darbu – komentē savas veiksmes un neveiksmes stundas gaitā, orientējoties uz saprotamu, pareizu stāstījumu. Vajadzētu panākt, lai katrs stundas gaitā būtu izteicies par matemātikas tēmu, lietojot atbilstošus vārdus un izteicienus.

Skolotāja pašnovērtējums: Izdara secinājumus par skolēnu spēju apgūt jaunu terminoloģiju, kā arī stundas atbilstību skolēnu matemātiskajām un valodas spējām un metožu efektivitāti.

Read the text. Prepare to ask and answer questions about it.

Who Invented Algebra?

Algebra is named after the Arabic word "*al-jabr*" from the title of a book written by the Arabic scholar, Muhammad ibn Musa al-Khwarizmi in 820.

Al-Khwarizmi lived approximately from 780 to 850 in the city of Khorezm (or Khwarezm), situated in the present-day Uzbekistan. He is considered by many to be the "father of algebra" and one of the greatest mathematicians of all time.

Al-Khwarizmi is famous for his general approach to problem solving. He was the first to solve linear and quadratic equations using general methods. Before him, each particular situation was solved separately, without a general description or plan. Some of his techniques for solving quadratic equations are still in use today.

The term "al-jabr" means ‘the reunion’ or ‘the equating’ and refers to the rearranging and combining of terms when solving equations.

Through his writings, the decimal system and the use of zero were introduced from India to the west. We can also thank him for the word ‘algorithm’ (which means a detailed, precise plan for solving a problem) – it is derived from the name Al-Khwarizmi.

Solving Equations

Teacher's Notes

An ***equation*** is an equality that contains one or more variables and that may be true or false depending on the values of these variables.

The values of variables for which the equality becomes true are called ***solutions*** or ***roots***. To solve an equation means to find all roots or to show that the equation has no roots at all.

For example, the root of the equation $2^x = 8$ is 3 (because $2^3 = 8$) but the equation $x^2 + 1 = 0$ has no roots (because the square of any real number cannot be negative).

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Questions to ask:

1. What is an equation? (Some possible answers – an equality with variables; an equality with unknown numbers; an equality that has to be solved – how? – the values of variables must be found, etc.)
2. What is a root of an equation? (A number with which the equality is correct/true; a value of *x* for which the equality is true etc.)
3. Why is Al-Khwarizmi famous? (He invented methods for solving equations that are still used nowadays; introduced decimal system and zero to Europe etc.)
4. What is important about his work? (He was the first to use general methods, to give a plan for solving equations, instead of considering each case separately; also answers to the previous questions.)

Solving Equations

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For example, the root of the equation $2^x = 8$ is 3 (because $2^3 = 8$) but the equation $x^2 + 1 = 0$ has no roots (because the square of any real number cannot be negative).

Example 1

Solve the linear equation $8(x-3)+5=-2(3x-1)$

Step 1. Open the brackets on both sides : $8x-24+5=-6x+2$

Step 2. Now, carry all terms containing *x* to the left side, and all constant terms to the right side of the equation. Remember that a term changes its sign to the opposite when carried across the equality sign: $8x+6x=2+24-5$

Step 3. Combine the like terms on both sides: $14x=21$

Step 4. Finally, divide both sides by 14 and simplify the fraction: $x=\frac{21}{14}=\frac{3}{2}=1.5$

You have just performed Algebra! Step 2 and Step 3 are what Al-Khwarizmi's word 'al-jabr' referred to.

Example 2

A special form of an algebraic equation is **proportion**. It is used to compare two ratios or make equivalent fractions. A proportion is typically written in the form $\frac{a}{b}=\frac{c}{d}$. The terms *a* and *d* are called the **extremes,** and *b* and *c* are called the **means**. When solving equations given in the form of proportion, we can apply the property of proportion:

the product of the extremes equals the product of the means, or $ad=bc$.

You probably remember this property as 'the diagonal products are equal'.

Now, let us solve the proportion $\frac{3x-1}{2}=\frac{5-2x}{7}$.

1. According to the property of proportion, $(3x-1)\cdot 7=(5-2x)\cdot 2$

2. Opening the brackets, $21x-7=10-4x$.

3. Rearranging and combining the terms, $21x+4x=10+7$; $25x=17$; $x=\frac{17}{25}=0.68$.

Example 3

Solve the **quadratic equation** $3x^2 - 2x - 5 = 0$.

Recall that to solve the equation $ax^2 + bx + c = 0$, the quadratic formula $x = \frac{-b \pm \sqrt{D}}{2a}$ is used where D is the **discriminant** $D = b^2 - 4ac$.

If the discriminant is positive then the equation has two roots; if D = 0, then one root, and if it is negative, then the equation has no roots.

First, determine the coefficients: $a = 3, \quad b = -2, \quad c = -5$.

The discriminant $D = (-2)^2 - 4 \cdot 3 \cdot (-5) = 4 + 60 = 64$; as the discriminant is positive, the equation has two roots: $x = \frac{2 \pm \sqrt{64}}{6} = \frac{2 \pm 8}{6}; \quad x_1 = \frac{2+8}{6} = \frac{10}{6} = \frac{5}{3} = 1\frac{2}{3}; \quad x_2 = \frac{2-8}{6} = \frac{-6}{6} - 1$.

Example 4

Solve the exponential equation $25^{x-4} = 5^{x+3}$.

As $25 = 5^2$, the equation can be rewritten as $\left(5^2\right)^{x-4} = 5^{x+3}$; then $5^{2(x-4)} = 5^{x+3}$ or $5^{2x-8} = 5^{x+3}$.

If the powers are equal and the bases are equal, then the exponents must also be equal:

$2x - 8 = x + 3$

$2x - x = 3 + 8$

$x = 11$

Example 5

Solve the exponential equation $3^{x+2} - 2 \cdot 3^x = 567$.

Factorise the first term: $3^x \cdot 3^2 - 2 \cdot 3^x = 567$

Factorise the left side by carrying out the common factor: $3^x \cdot \left(3^2 - 2\right) = 567$.

Perform the operations in brackets: $3^x \cdot 7 = 567$

and divide both sides by 7: $3^x = 81$.

Now, as $81 = 3^4$,

$3^x = 3^4$ and $x = 4$.

Exercise

Solve the equations:

1. $\frac{2}{3}x = 10 - x$
2. $5(x-2) = 3(x+4)$
3. $\frac{2x-3}{6} = \frac{3x+1}{4}$
4. $x^2 - 5x + 12 = 0$
5. $4x^2 + 12x + 9 = 0$
6. $6^{2x+5} = \frac{1}{36}$
7. $10^{x^2} = 10^{3x}$
8. $2^{3x} - 2^{3x-2} = 48$

Exercise problem solutions

Teacher's Notes

1.

$\frac{2}{3}x = 10 - x$

$\frac{2}{3}x + x = 10$

$1\frac{2}{3}x = 10$

$\frac{5}{3}x = 10$

$x = \frac{10 \cdot 3}{5} = 6$

2.

$5(x-2) = 3(x+4)$

$5x - 10 = 3x + 12$

$5x - 3x = 12 + 10$

$2x = 22$

$x = 11$

3.

$\frac{3x+1}{4} = \frac{2x-3}{6}$

$6(3x+1) = 4(2x-3)$

$18x + 6 = 8x - 12$

$18x - 8x = -12 - 6$

$10x = -18$

$x = -\frac{18}{10} = -1.8 \left(\text{or } x = -\frac{18}{10} = -\frac{9}{5} = -1\frac{4}{5} \right)$

4.

$x^2 - 5x + 12 = 0$

$a = 1; \quad b = -5; \quad c = 12;$

$D = (-5)^2 - 4 \cdot 1 \cdot 12 = 25 - 48 = -23$

The discriminant is negative, therefore the equation **has no roots**.

5.

Method 1:

$4x^2 + 12x + 9 = 0$

$a = 4; \quad b = 12; \quad c = 9;$

$D = 12^2 - 4 \cdot 4 \cdot 9 = 144 - 144 = 0$

The discriminant equals 0m therefore the equation has one root:

$x = \frac{-12 \pm 0}{8} = -1.5.$

Method 2:

Note that the left side is a perfect square:

$(2x+3)^2 = 0$

Therefore

$2x + 3 = 0$

$2x = -3$

$x = -1.5$

6.

$6^{2x+5} = \frac{1}{36}$

$6^{2x+5} = 6^{-2}$

$2x + 5 = -2$

$2x = -7$

$x = -3.5$

7.

$10^{x^2} = 10^{3x}$

$x^2 = 3x$ Note that both sides must **not** be divided by x

$x^2 - 3x = 0$

$x(x-3) = 0$

The product equals 0 if and only if one of the factors equals 0, therefore

$x_1 = 0 \quad x_2 = 3.$

8.

$2^{3x} - 2^{3x-2} = 48$

$2^{3x} - 2^{3x} \cdot 2^{-2} = 48$

$2^{3x} \cdot \left(1 - \frac{1}{4}\right) = 48$

$2^{3x} \cdot \frac{3}{4} = 48$

$2^{3x} = \frac{48 \cdot 4}{3}$

$2^{3x} = 64$

$2^{3x} = 2^6$

$3x = 6$

$x = 2$

Stundas piemērs

English For Mathematics
Lesson 3
Proportions in Geometry (Proporcijas ģeometrijā)

Mērķis:

1. Iepazīstināt skolēnus ar angļu valodas lietojumu matemātikā, it īpaši ģeometrijā.
2. Atkārtot vienkāršas ģeometriskas pamatsakarības līdzīgos trijstūros.
3. Sniegt priekšstatu par zelta griezumu.

Skolēnam sasniedzamais rezultāts:

1. Saprot vienkāršu matemātisku tekstu par trijstūru līdzību angļu valodā.
2. Prot izklāstīt vienkāršu ģeometrisku uzdevumu risinājumu angļu valodā.

Nepieciešamie resursi:

1. Teksti „Similar Triangles", „The Golden Ratio" ar uzdevumiem risināšanai
2. Jauno vārdu (matemātikas terminu) saraksts ar tulkojumiem, vārdnīca.
3. Lineāls, zīmulis, papīrs zīmēšanai.

Mācību metodes:

1. Darbs ar tekstu.
2. Jautājumi un atbildes.
3. Uzdevumu risināšana

Mācību procesa organizācijas formas:

1. Frontāls darbs.
2. Diskusija.

Vērtēšana: Skolotāja frontāls mutisks vērtējums.

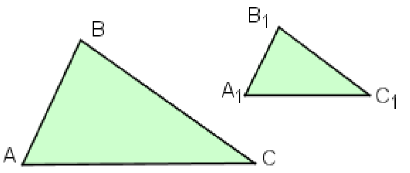
Stundas piemērs

Stundas gaita: Līdzīgi iepriekšējām, tā būs atkarīga no konkrētās klases spējām un izpratnes līmeņa. Rezervē (īpaši spējīgiem skolēniem, kas strādā ātri) piedāvāts pētnieciska rakstura darbs 'Zelta griezums regulārā piecstūrī'.

Skolotāja darbība	Skolēnu darbība
Metode (laiks)	
Ierosināšana (5 min). Skolotājs lūdz skolēnus saviem vārdiem paskaidrot, kas ir proporcija (skat 2. stundas materiālu), un kas ir līdzīgi trijstūri (jauns).	Skolēni īsi paskaidro, kas ir proporcija. Apraksta, kādi trijstūri ir līdzīgi.
Metode (laiks)	
Apjēgšana. (25min) 1. Skolotājs uzaicina izlasīt tekstu ***Similar Triangles*** un pārliecināties, ka tas atbilst skolēnu stāstītajam. 2. Uzdevumu risināšana. Skolotājs uzaicina skolēnus lasīt un veikt 4. vingrinājuma uzdevumus (pēc skolotāja ieskatiem un izvēles, visus vai tikai dažus). 3. Skolotājs aicina skolēnus iepazīties ar tekstu ***The Golden Ratio.***	1. Skolēni lasa tekstu. Matemātiskās sakarības tekstā var nelasīt skaļi. 2. Katram izvēlētajam uzdevumam: skolēni lasa vingrinājuma uzdevumu un veido zīmējumu; viens skolēns pie tāfeles, klase kopīgi ierosina, kā risināt. 3. Skolēni lasa tekstu. Spējīgākie matemātiķi var paši atrisināt vienādojumu, kurš tajā dots, un īsi iepazīstināt klasi ar atrisinājumu.
Metode (laiks)	
Refleksija. (10 *min*) Skolotājs uzaicina skolēnus izmantot tekstā ***Task: Drawing the Golden Spiral*** doto informāciju un uzzīmēt zelta spirāli, kā arī, vērojot attēlus, komentēt redzēto. Noslēgumā īsa saruna, novērtējot angļu valodas ietekmi uz matemātikas izpratni. Cik lielā mērā var apgalvot, ka matemātika pati par sevi jau ir universāla valoda un tāpēc saprotama (vai nesaprotama) neatkarīgi no saziņas valodas?	Skolēni zīmē spirāli, sekojot tekstā dotajiem norādījumiem un attēlam. Vērojot zelta spirālei līdzīgos dabas veidojumus (arī attēlus iepriekšējā tekstā), izsaka savus komentārus (piemēram, ievēro, ka precizitāte ir samērā aptuvena – vai gluži pretēji, uzskata, ka sakritība ir pārsteidzoša).

Skolotāja pašnovērtējums: Secinājumi par stundas atbilstību skolēnu matemātiskajām spējām un valodas zināšanām. Gala secinājumi par matemātikas 'valodas' universālu saprotamību.

Similar Triangles



Two triangles are called ***similar*** if they have the same shape but not necessarily the same size. More precisely, **all the corresponding angles are equal and all the corresponding sides are in proportion.**

That is: $\triangle ABC \sim \triangle A_1B_1C_1$ if $\angle A = \angle A_1$, $\angle B = \angle B_1$, $\angle C = \angle C_1$ and $\frac{AB}{A_1B_1} = \frac{BC}{B_1C_1} = \frac{AC}{A_1C_1}$.

For two triangles to be similar, it is sufficient to know that **two pairs of corresponding angles are equal.**

When writing down similar triangles, always make sure that the corresponding points are written in the same order in both triangles!

Exercise 1

Show that the triangles ABC and DEF are similar (check all angles and all sides) if
$\angle A = 106°, \angle B = 34°, \angle E = 106°, \angle F = 40°$;
AC = 4.4cm, AB = 5.2 cm, BC = 7.6 cm, DE = 15.6cm, DF = 22.8cm, EF = 13.2cm.
Write the similarity in the correct point order: $\triangle ABC \sim \triangle$.............

Exercise 2

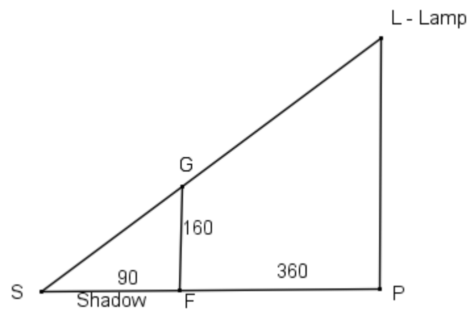
For the following questions, first draw a sketch of similar triangles; then write the proportion of corresponding sides using the letters; then use numbers to calculate the required answers..

(a) $\triangle ABC \sim \triangle KMN$; AB = 4cm, BC = 5cm, CA = 7cm; $\frac{KM}{AB} = 2.5$. Find the sides of $\triangle KMN$.

(b) $\triangle DEF \sim \triangle KMN$; find the ratio of sides and calculate both perimeters given that DE = 6cm, EF = 10.5cm, KM = 12cm, KN = 14cm.

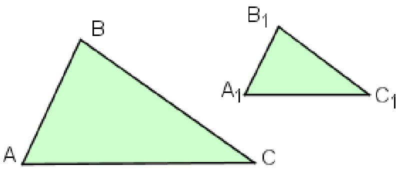
Exercise 3

A girl 160 cm tall, stands 360 cm from a lamp post at night. Her shadow from the light is 90 cm long. How high is the lamp post?



Similar Triangles

Teacher's Notes



Two triangles are called ***similar*** if they have the same shape but not necessarily the same size. More precisely, **all the corresponding angles are equal and all the corresponding sides are in proportion.**

That is: $\triangle ABC \sim \triangle A_1B_1C_1$ if $\angle A = \angle A_1$, $\angle B = \angle B_1$, $\angle C = \angle C_1$ and $\frac{AB}{A_1B_1} = \frac{BC}{B_1C_1} = \frac{AC}{A_1C_1}$.

For two triangles to be similar, it is sufficient to know that **two pairs of corresponding angles are equal.** (E.g., $\angle A = \angle A_1$, $\angle B = \angle B_1$ or $\angle B = \angle B_1$, $\angle C = \angle C_1$. That means the third pair of angles are also equal – because the sum of all angles in a triangle is always 180°, so the third angle is determined if the other two are known).

When writing down similar triangles, always make sure that the corresponding points are written in the same order in both triangles!

Exercise 1

Show that the triangles ABC and DEF are similar (check all angles and all sides) if

$\angle A = 106°, \angle B = 34°, \angle E = 106°, \angle F = 40°$;

AC = 4.4cm, AB = 5.2 cm, BC = 7.6 cm, DE = 15.6cm, DF = 22.8cm, EF = 13.2cm.

Write the similarity in the correct point order: $\triangle ABC \sim \triangle$.............

Find the third angle in each triangle: $\angle C = 180° - (106° + 34°) = 40°$; $\angle D = 180° - (106° + 40°) = 34°$;

so the corresponding angles are $A \leftrightarrow E,\ B \leftrightarrow D,\ C \leftrightarrow F$. $\triangle ABC \sim \triangle EDF$

Checking the side proportion: $\frac{AB}{ED} = \frac{BC}{DF} = \frac{AC}{EF}$ is true because $\frac{5.2}{15.6} = \frac{7.6}{22.8} = \frac{4.4}{13.2}\left(= \frac{1}{3}\right)$.

Exercise 2

For the following questions, first draw a sketch of similar triangles; then write the proportion of corresponding sides using the letters; then use numbers to calculate the required answers..

(a) $\triangle ABC \sim \triangle KMN$; AB = 4cm, BC = 5cm, CA = 7cm; $\frac{KM}{AB} = 2.5$. Find the sides of $\triangle KMN$.

$\frac{KM}{AB} = \frac{MN}{BC} = \frac{KN}{AC}$ or $\frac{KM}{4} = \frac{MN}{5} = \frac{KN}{7} = 2.5$; from here

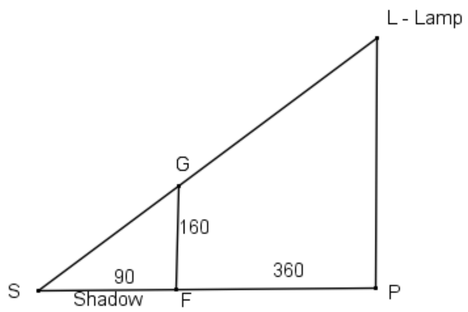
$KM = 2.5 \cdot 4 = 10;\quad MN = 2.5 \cdot 5 = 12.5;\quad KN = 2.5 \cdot 7 = 17.5$

(b) $\triangle DEF \sim \triangle KMN$; find the ratio of sides and calculate both perimeters given that DE = 6cm, EF = 10.5cm, KM = 12cm, KN = 14cm.

$\frac{KM}{DE} = \frac{MN}{EF} = \frac{KN}{DF}$ becomes $\frac{12}{6} = \frac{MN}{10.5} = \frac{14}{DF}$; the ratio is $\frac{12}{6} = 2$.

Solving separately, $\frac{MN}{10.5} = 2 \Rightarrow MN = 21$; $\frac{14}{DF} = 2 \Rightarrow DF = 7$.

Exercise 3



A girl 160 cm tall, stands 360 cm from a lamp post at night. Her shadow from the light is 90 cm long. How high is the lamp post?

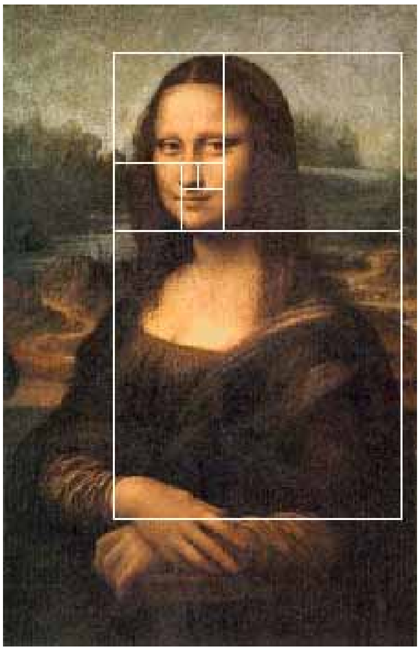
$\triangle SLP \sim \triangle SGF$ (similar by two angles as they have a common angle S and a right angle at F and P)

$\frac{LP}{GF} = \frac{SP}{SF}$ (no need for the third pair of sides); the ratio $\frac{LP}{160} = \frac{90+360}{90} = \frac{450}{90} = 5$;

$LP = 5 \cdot 160 = 800$ cm or 8 m.

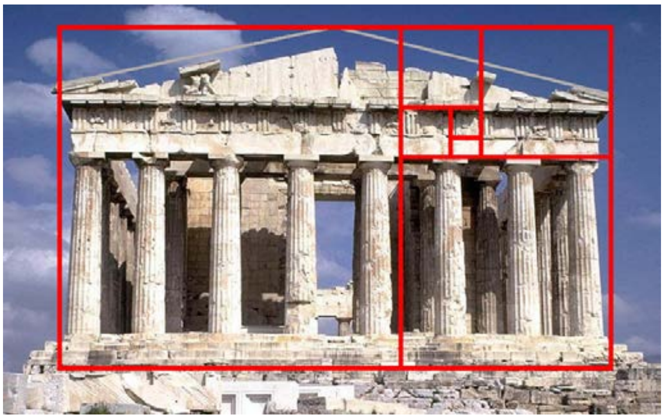
Read the text. Try to solve the Golden Ratio equation.

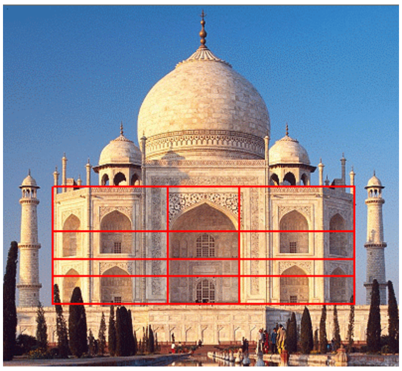
The Golden Ratio



The Golden Ratio or Golden Section has provided much inspiration to artists and architects at all times. The sculptors in ancient Greece believed that the proportions of the ideal human body are based on the Golden Ratio, as did Leonardo Da Vinci. Famous buildings, like the Parthenon in Greece, Notre Dame in Paris, Taj Mahal in India, even the pyramids of Egypt have the Golden Ratio applied in their design. It is considered the ratio most pleasing to the eye. The Renaissance artists in the time of Leonardo Da Vinci knew it as the Divine Proportion.

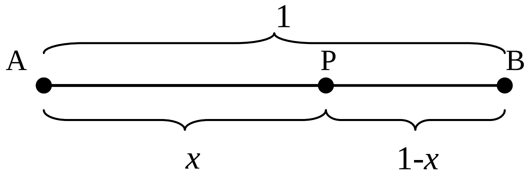
Ratios that are very close to the golden ratio can be found in the nature quite often.





What exactly is the Golden Ratio?

Suppose we have a line segment AB and a point P dividing it into two parts.
If the ratio of the two parts is equal to the ratio of the whole segment and its longer part – that is, $\frac{AP}{PB}=\frac{AB}{AP}$ – then it is called the **golden ratio**.



Let AB = 1 unit, and let the longer part be AP = *x*; then BP = 1 - *x*.

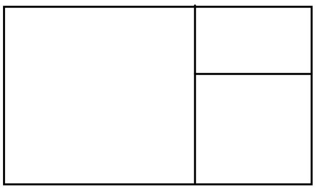
Then the proportion $\frac{AP}{PB}=\frac{AB}{AP}$ becomes $\frac{x}{1-x}=\frac{1}{x}$. The positive root of this equation is $x=\frac{\sqrt{5}-1}{2}$.

The Golden Ratio is $\frac{1}{x}=\frac{2}{\sqrt{5}-1}=\frac{1+\sqrt{5}}{2}=1.6180339...$ and it is denoted by the Greek letter Φ (Phi, pronounced [fai] or [fi:]).

This number has many interesting properties; for example, its square can be obtained by simply adding 1 to it: $\Phi^2=\Phi+1$.

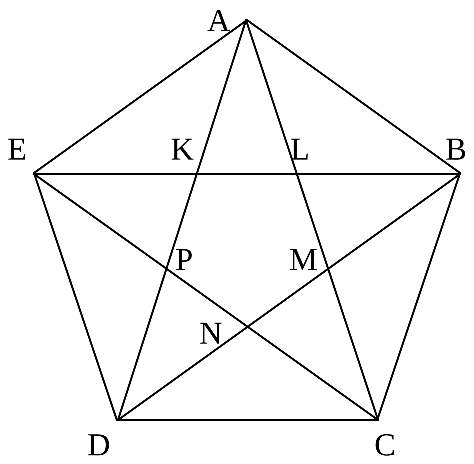
In the **golden rectangle** the ratio of the side lengths is Φ. If we cut off a square, the remaining part is again a golden ratio rectangle.

If the process is repeated many times, we can obtain **the golden spiral**.



Pentagram and the Golden Ratio: Investigation

Investigate the regular pentagon ABCDE where K, L, M, N, P are the intersection points of its diagonals.



1. What is the angle at each vertex of the pentagon?
2. Find all the angles in the triangles formed.
3. Find as many pairs of similar triangles as you can.
4. Use the triangles ABK and KAL to show that the point L splits the line segment BK in the golden ratio, that is, that $\frac{KL}{LB} = \frac{LB}{BK}$.
5. Can you find more golden sections in this pentagon?

Task: Drawing the Golden Spiral

Let us draw the golden spiral, starting with a golden rectangle.

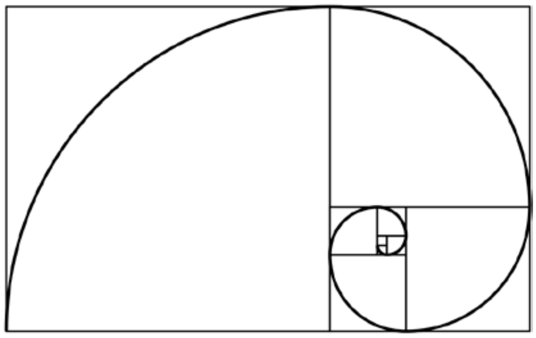
First, we need to choose the side lengths of the first rectangle – choose the shorter side and use the number Φ to calculate the longer side (for example, if the shorter side is 10 cm, then the longer side will be $10\Phi \approx 10 \cdot 1.618 \approx 16.2$ cm. (Make sure your drawing is as large as possible.)

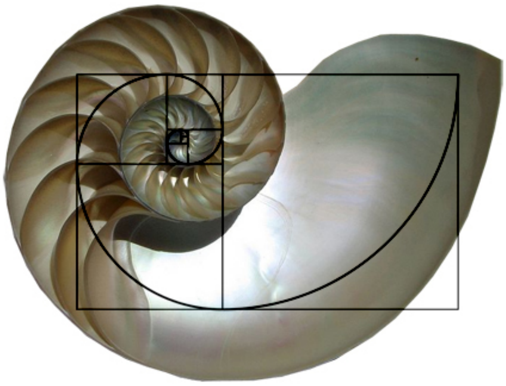
When the rectangle has been drawn, divide it into a square and another rectangle.

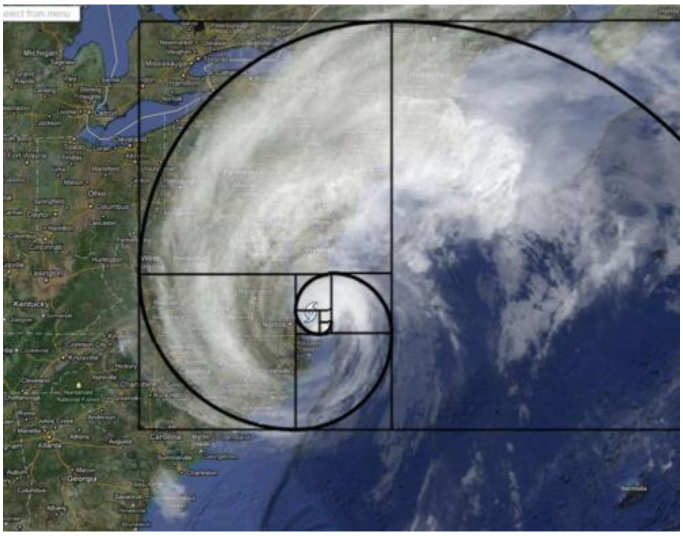
Do the same in the new rectangle, and repeat as many times as you can – the rectangles will soon become very small!

Make sure the squares lie in a spiral pattern.

Finally, draw a smooth spiral through the vertices of the squares.







Glossary

English – Latvian

addition	-	saskaitīšana
to add	-	saskaitīt, pieskaitīt
angle	-	leņķis
base	-	[pakāpes] bāze
coefficient	-	koeficients
decimal	-	*n* decimāldaļa; *a* decimāls
degree	-	grāds
difference	-	starpība
division	-	dalīšana
to divide [by]	-	dalīt [ar]
equality	-	vienādība
equation	-	vienādojums
exponent	-	kāpinātājs
expression	-	izteiksme
factor	-	reizinātājs
to factorise	-	sadalīt reizinātājos
fraction	-	daļskaitlis, daļa
general	-	vispārīgs
golden ratio	-	zelta griezums
identity	-	identitāte
like terms	-	līdzīgie locekļi
to combine like terms	-	savilkt līdzīgos saskaitāmos
linear	-	lineārs
line segment	-	taisnes nogrieznis
multiplication	-	reizināšana
to multiply	-	reizināt
particular	-	atsevišķs
pentagon	-	piecstūris
power	-	pakāpe (attiecas uz visu izteiksmi a^b kopā)
problem	-	problēma, [matemātisks] uzdevums
product	-	reizinājums
proportion	-	proporcija
quadratic	-	kvadrāt- (par vienādojumiem, funkcijām)
quotient	-	dalījums
real numbers	-	reālie skaitļi
rectangle	-	taisnstūris
root	-	sakne
solution	-	atrisinājums
square	-	[skaitļa] kvadrāts ; kvadrāts (ģeometriska figūra)
square root	-	kvadrātsakne
subtraction	-	atņemšana
to subtract	-	atņemt

sum	-	summa
term	-	1. termins 2. [izteiksmes] loceklis; saskaitāmais
the extremes	-	malējie locekļi (proporcijā)
the means	-	vidējie locekļi (porporcijā)
triangle	-	trijstūris
similar triangles	-	līdzīgi trijstūri
value	-	vērtība
variable	-	mainīgais
vertex (pl. vertices)	-	virsotne

Latvian –English

atņemšana	-	subtraction
atņemt	-	to subtract
atrisinājums	-	solution
atsevišķs	-	particular
bāze	-	base
dalījums	-	quotient
dalīšana	-	division
dalīt [ar]	-	to divide [by]
daļskaitlis, daļa	-	fraction
decimāldaļa	-	decimal
grāds	-	degree
identitāte	-	identity
izteiksme	-	expression
kāpinātājs	-	exponent
koeficients	-	coefficient
kvadrāt- (par vienādojumiem, funkcijām)	-	quadratic
kvadrāts (skaitļa; ģeometriska figūra)	-	square
kvadrātsakne	-	square root
leņķis	-	angle
lineārs	-	linear
līdzīgi trijstūri	-	similar triangles
līdzīgie locekļi	-	like terms
loceklis (izteiksmes); saskaitāmais	-	term
mainīgais	-	variable
pakāpe (par izteiksmi a^b)	-	power
piecstūris	-	pentagon
problēma, [matemātisks] uzdevums	-	problem
proporcija	-	proportion
proporcijas malējie locekļi	-	the extremes
proporcijas vidējie locekļi	-	the means

reālie skaitļi	-	real numbers
reizinājums	-	product
reizināšana	-	multiplication
reizināt	-	to multiply
reizinātājs	-	factor
sadalīt reizinātājos	-	to factorise
sakne	-	root
saskaitīšana	-	addition
saskaitīt, pieskaitīt	-	to add
savilkt līdzīgos saskaitāmos	-	to combine like terms
starpība	-	difference
summa	-	sum
taisnes nogrieznis	-	line segment
taisnstūris	-	rectangle
trijstūris	-	triangle
vērtība	-	value
vienādība	-	equality
vienādojums	-	equation
virsotne	-	vertex (pl. vertices)
vispārīgs	-	general
zelta griezums	-	golden ratio